Probability Rules Cheat Sheet

Your comprehensive guide to Probability Rules

Descriptive Statistics

Sample Mean:

$$\bar{x} = \frac{\sum x_i}{n}$$

Population Mean:

$$\mu = \frac{\sum x_i}{N}$$

Population Variance:

$$\sigma^2 = \frac{\sum_{i=1}^{N}(x_i - \mu)^2}{N}$$

Sample Variance:

$$s^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1}$$

Sample Standard Deviation:

$$s = \sqrt{\frac{\sum(x_i - \bar{x})^2}{n-1}}$$

Population Standard Deviation:

$$\sigma = \sqrt{\frac{\sum(x_i - \mu)^2}{N}}$$

Percentile: Value below which a certain percentage of data falls
Quartiles: Q1 (25th percentile), Q2 = Median (50th percentile), Q3 (75th percentile)
Interquartile Range (IQR): $Q_3 - Q_1$
Range: Maximum - Minimum

Distribution Shapes: - Symmetric: Mean $\approx$ Median - Right-skewed: Mean $>$ Median (tail extends to the right) - Left-skewed: Mean $<$ Median (tail extends to the left) - Outliers affect the mean more than the median

Probability Rules

Basic Probability Concepts

Probability Definition:

$$P(A) = \frac{\text{Number of favorable outcomes}}{\text{Total number of outcomes}}$$

Properties: - $0 \leq P(A) \leq 1$ - $P(\emptyset) = 0$ (impossible event) - $P(S) = 1$ (certain event, where S is sample space)

Example: Rolling a fair die, probability of getting an even number:

$$P(\text{even}) = \frac{3}{6} = \frac{1}{2} = 0.5$$

Practice: What is the probability of drawing a face card from a standard deck?
Answer: $P(\text{face card}) = \frac{12}{52} = \frac{3}{13}$

Complement Rule

Formula:

$$P(A^c) = 1 - P(A)$$

Alternative notation: $P(A') = 1 - P(A)$

Explanation: The probability that event A does not occur.

If the probability that Anya will graduate is 0.9, then the probability she will not graduate is:

$$P(\text{not graduate}) = 1 - 0.9 = 0.1$$

If $P(\text{rain}) = 0.3$, what is $P(\text{no rain})$?
Answer: $P(\text{no rain}) = 1 - 0.3 = 0.7$

Addition Rules

General Addition Rule (For Any Two Events)

Formula:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Explanation: We subtract $P(A \cap B)$ to avoid double-counting the overlap.

In a class of 24 students, 10 are girls, 11 are A students, and 6 are girls who are A students.
Probability of selecting a girl or an A student:

$$P(\text{girl or A}) = \frac{10}{24} + \frac{11}{24} - \frac{6}{24} = \frac{15}{24} = 0.625$$

Addition Rule for Mutually Exclusive Events

Formula:

$$P(A \cup B) = P(A) + P(B)$$

Condition: $P(A \cap B) = 0$ (events cannot occur simultaneously)

Probability of rolling a 2 or 6 on a die:

$$P(2 \text{ or } 6) = \frac{1}{6} + \frac{1}{6} = \frac{2}{6} = 0.333$$

A bag contains 4 red, 3 blue, and 2 green marbles. What's the probability of drawing a red or green marble?
Answer: $P(\text{red or green}) = \frac{4}{9} + \frac{2}{9} = \frac{6}{9} = \frac{2}{3}$

Multiplication Rules

Multiplication Rule for Dependent Events

Formula:

$$P(A \cap B) = P(A) \times P(B|A)$$

Alternative: $P(A \cap B) = P(B) \times P(A|B)$

Drawing two red cards without replacement from a standard deck:

$$P(\text{red and red}) = \frac{26}{52} \times \frac{25}{51} = 0.245$$

Multiplication Rule for Independent Events

Formula:

$$P(A \cap B) = P(A) \times P(B)$$

Condition: Events are independent if $P(A|B) = P(A)$

Drawing two red cards with replacement:

$$P(\text{red and red}) = \frac{26}{52} \times \frac{26}{52} = 0.25$$

Two fair coins are flipped. What's the probability of getting two heads?
Answer: $P(HH) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

Conditional Probability

Formula:

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Condition: $P(B) > 0$

Explanation: The probability of event A occurring given that event B has occurred.

In a group of 100 people, 60 are employed and 40 are unemployed. Of the employed, 45 are satisfied with their job.
What's the probability someone is satisfied given they are employed?

$$P(\text{satisfied}|\text{employed}) = \frac{45}{60} = 0.75$$

A card is drawn from a deck. Given that it's red, what's the probability it's a heart?
Answer: $P(\text{heart}|\text{red}) = \frac{13}{26} = \frac{1}{2}$

Set Operations and Probability

Union (OR): - Symbol: $A \cup B$ - Meaning: Event A OR event B (or both) occurs - Formula: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Intersection (AND): - Symbol: $A \cap B$ - Meaning: Both events A AND B occur - Formula: $P(A \cap B) = P(A) \times P(B|A)$

Complement (NOT): - Symbol: A^c or A' - Meaning: Event A does NOT occur - Formula: $P(A^c) = 1 - P(A)$